Name: _____

- This packet is due at **5:00 pm PDT on Wednesday, May 5**.
- Neatly fill out this packet either directly or on your own paper. Then upload your answers to Canvas. Please clearly number each problem and keep them in order.
- For each problem in this packet, **organize and show all of your work and justification** for your answer.
- Make sure that it is clear what your final answer is. This may mean boxing/circling it or it may mean doing neither.
- If you are required to draw a graph of a function, each axis must be labeled and given a scale.
- I will not be grading every individual problem, but answers will be available on Canvas after the due date.
- I am happy to help you however I can. Here are some options:
  - Come to my office hours (see syllabus).
  - Email me directly at gknapp4@uoregon.edu

1. Use the ratio test to determine whether $\sum_{n=1}^{\infty} a_n$ converges, where a_n is given in the following problems. State if the ratio test is inconclusive.

   (a) $a_n = \dfrac{7^n}{n!}$

   (b) $a_n = \dfrac{n^7}{(-3)^n}$

$$\lim_{n\to\infty} \left| \frac{(n^7 + \text{L.D.T})(-3)^n}{n^7 \quad (-3)^n \cdot (-3)} \right| = \left| 1 - \frac{1}{3} \right| < 1$$

(c) $a_n = \dfrac{(2n)!}{n^2}$

$$(2^5)^n = 2^{5n}$$

(d) $a_n = \dfrac{2^{5n}(n!)^3}{(3n)!}$

$$\lim_{n\to\infty} \frac{2^{5(n+1)} \cdot \left((n+1)!\right)^3}{(3(n+1))!} \cdot \frac{(3n)!}{2^{5n}(n!)^3}$$

$(3n+3)! = (3n+3)\cdot(3n+2)!$
$= (3n+3)(3n+2)\cdot(3n+1)!$
$= (3n+3)(3n+2)(3n+1)\cdot(3n)!$

$$\lim_{n\to\infty} \frac{2^{5n}\cdot 2^5}{2^{5n}} \cdot \frac{\left[(n+1)!\right]^3}{(n!)^3} \cdot \frac{(3n)!}{(3n+3)!}$$

$$\lim_{n\to\infty} 2^5 \cdot \frac{(n+1)^3 (n!)^3}{(n!)^3} \cdot \frac{(3n)(3n-1)\cdots 2\cdot 1}{(3n+3)(3n+2)(3n+1)(3n)\cdots 2\cdot 1}$$

$$\lim_{n\to\infty} \frac{2^5 \cdot (n+1)^3}{(3n+3)(3n+2)(3n+1)} = \lim_{n\to\infty} \frac{2^5 \cdot (n^3 + \text{L.D.T.})}{27n^3 + \text{L.D.T}}$$

$$= \frac{2^5}{27} > 1$$

2. Use the root test to determine whether $\sum_{n=1}^{\infty} a_n$ converges where a_n is as follows.

(a) $a_n = \dfrac{(-2)^n}{n}$

(b) $a_k = \dfrac{k^e}{e^k}$

(c) $a_k = \dfrac{1}{(1+\ln k)^k}$

$\lim_{k\to\infty} \dfrac{1}{1+\ln(k)} < 1$

By the root test, $\sum_{k=2}^{\infty} \dfrac{1}{(1+\ln(k))^k}$ converges

(d) $a_n = \dfrac{(\ln(1+\ln n))^n}{(\ln n)^n}$

$\dfrac{\infty}{\infty}$ L'Hospital

$\lim_{n\to\infty} \dfrac{\ln(1+\ln(n))}{\ln(n)} =$

Picking a Comparison

(1) Decide if converges / diverges

(2) If convergence,

- D.C.T. Find upper bd. that you know converges

- L.C.T. Find something that you know converges which grows about as fast

↳ list of fn. growth rates

If diverges

- D.C.T. Find lower bd. that you know diverges

- L.C.T. Find something that you know diverges which grows about as fast

Exs: $\frac{\text{Polynomial}}{\text{polynomial}} \sim \frac{\text{leading term}}{\text{leading term}}$

$$\frac{7n^3 - 3n + 1}{3n^8 + 2} \sim \frac{7n^3}{3n^8} = \frac{7}{3n^5}$$